Lattice Gauge Theory for Physics beyond the Standard Model

Lattice QCD Executive Committee
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Abstract

The implications of lattice field theory for particle physics go far beyond the traditional studies of low energy QCD phenomenology which are currently the major focus of the field. New strongly coupled field theories may well be discovered at LHC scales; for these the lattice will be the major tool. Also supersymmetry is important to many high energy models so their phase structure and spectrum are needed, but they are only beginning to be explored on the lattice and the fundamental connections to the lattice regulator remain wide open. While opportunities abound, there are also theoretical challenges for the lattice approach. We still do not have a non-perturbative regulator for chiral gauge theories, such as those relevant to the weak interactions, without which the very existence of the standard model can be called into question. Sign problems plague numerical approaches to many interesting phenomena such as color superconductivity. All of these questions insure a healthy and challenging future for lattice field theory.

1 Introduction

The LHC era is likely to expose new non-perturbative physics beyond the QCD sector of the standard model. Physicists exploring candidate models have developed an overwhelming array of possible scenarios. To really understand the options and make a definite discrimination between experimental signatures requires investigations of lattice field theory beyond the realm of QCD [1]. Fortunately, the coming era coincides with increasing access to Petascale hardware. Thus lattice studies of a large range of non-perturbative physics in the growing landscape of theories will become increasingly important and realistic. This exploratory approach has proven very useful in lower dimensions for condensed matter physics where the computational requirements are generally $O(10^3)$ to $O(10^6)$ less demanding. In the near future, a simple consequence of the availability of 10^6 Gigaflop/s platforms will make this exploratory approach also practical for 4-d quantum field theories. Of course, both the fundamental and technical challenges are significant and progress will depend on a balanced program of phenomenological surveys and continued work on new theoretical methods for lattice field theory.

There are 3 main scenarios envisioned for physics in the TeV energy range to be probed in the LHC era. Each implies challenges requiring the extension of lattice field theory beyond QCD.

- **Standard Higgs formulations of the Standard Model:** The first scenario is the discovery of the standard model Higgs with little hint of its origin. This is perhaps the least exciting; nevertheless, lattice QCD will continue to play its central role in high precision tests of the standard model, a topic dealt with in detail in the accompanying white paper on fundamental parameters. Precise calculations of hadronic matrix elements will continue to be needed, particularly if small discrepancies with the standard model begin to appear. Other related topics that have already received some attention are bounds on the mass of the Higgs boson and issues such as hadronic corrections to proton decay and the connection between electric dipole moments and strong CP violation. Additional topics include an accurate treatment of symmetry restoration in the early universe, and the question of whether the coupling of the Higgs to the top quark will involve appreciable non-perturbative physics.

- **SUSY field theories:** The second potential scenario involves the discovery of supersymmetry with its attendant zoo of new particles. In this case the need for lattice field theory to incorporate SUSY and to investigate its corresponding breaking pattern and vacuum structure will become paramount. This promises to be an extremely challenging but fruitful problem both because of the technical difficulties of formulating SUSY field theories on the lattice and because of the large range of possible SUSY field theories and breaking patterns.

- **New strong dynamics:** The third scenario is the discovery of a new strong dynamics and its interrelation with the structure of the standard model. This is of course an ideal situation for lattice field theory. The Higgs may well be most cleanly described as a composite arising in a new strongly coupled gauge field theory. The difficulties of traditional model building in this area require an exquisite interplay between this dynamics and the constraints on flavor physics, C and P violation, etc. Can these models generate the observed fermion masses? Can they resolve such issues as the huge ratios between the masses of neutrinos and the top quark? To unravel these phenomena may well demand a careful treatment of non-perturbative effects beyond the standard model, something for which the lattice is the most powerful tool we have.

Regardless of which of these scenarios plays out, it is important to emphasize that the investigation of quantum field theory using lattice tools can also explore phenomena well beyond the TeV range accessible at LHC energies. Many fundamental issues in quantum field theory require non-perturbative understanding. Indeed the lattice approach has already established a distinguished record in this regard. The phenomenon of confinement, crucial to the viability of QCD, while not proven analytically, has been convincingly demonstrated to be a property of the theory. The fact that chiral symmetry is spontaneously broken and is the explanation for the light nature of the pions is well verified in simulations. The lattice has also played a key role in our understanding of the Higgs mechanism, placing rather stringent bounds on the Higgs mass. Many other issues on the nature of the real space renormalization group, phase transitions at finite temperature, topological structure of the vacuum, and so on have been illuminated by lattice calculations.

Future topics should also include the strong/weak duality of the Maldacena AdS/CFT conjecture, model building methods of deconstruction from high dimensions, triviality and ultraviolet completion, the large N_c limit of Yang Mills theory (including QCD), and matrix model reductions. Particle physicists are only beginning to gain a deeper appreciation of the non-perturbative complexities of relativistic quantum theory. Lattice simulations will inevitably continue to play a major role in this broad enterprise.

2 Theoretical topics

We divide the topics into 4 broad categories, although this is admittedly somewhat arbitrary from the physical perspective. There are many interesting intersections and cross references which can be drawn between them. These overlapping areas are even more pronounced from the software and algorithmic standpoint. While there are fundamental challenges in developing new algorithms for an enlarged range of applications, on the basis of the SciDAC software infrastructure used by USQCD, it is important to introduce new gauge groups and new matter (Dirac or scalar) gauged in a variety of ways. Once this is done, we would have a rather coherent set of codes that would be capable of exploring a substantial range of lattice field theories. Indeed we recommend the extension of the notion of a shared tool box be included in future SciDAC software planning for this class of theories. The main constraint on this exploratory research remains convenient access to large computer resources for a broad spectrum of theorists to obtain results in months rather than years, even when the risk of failure is high for any specific model.

2.1 TeV scale strong coupling models

In the standard model, the cross section for the weak gauge boson scattering increases in the energy regime between the W boson and Higgs masses. Therefore, if the Higgs mass is significantly larger than the W boson mass, the scattering becomes non-perturbatively strong. This is an interesting signature which we expect to observe at the LHC: either a Higgs or other light particles which contribute as resonances to WW scattering or the W boson scattering cross section gets large. Theoretical models with strong W boson scattering have received a lot of attention in the literature. Examples are technicolor [2], Higgsless models [3], and extra-dimensional (Randall Sundrum) models [4]. In these models, physics at or near the TeV scale is non-perturbative and lattice methods are necessary to obtain quantitative predictions. For example, in these models the lattice would be useful to compute precision electroweak variables such conventional so-called S and T parameters: S measures momentum-dependent mixing of the electroweak gauge bosons and T the isospin breaking which splits the W and Z masses. At present the only other non-perturbative tools are qualitative in nature, including effective Lagrangians or ideas based on the AdS/CFT arguments as we discuss in Sec. 2.4.

In technicolor theories it is desirable to have "walking" [5]; that is the couplings and anomalous dimensions need to remain non-perturbatively large over a substantial range of energy scales. Examples of walking theories are known in supersymmetry and have been conjectured for non-supersymmetric theories. It is potentially very useful to verify this behavior explicitly on the lattice and to study the properties of such theories with regards to chiral symmetry breaking, flavor structure and precision electroweak consequences.

A primary question for walking gauge theories, especially non-supersymmetric ones, is the number of fermions required to achieve this behavior. Since walking arises near the transition from the chirally broken to the chirally symmetric vacuum as a function of the number of fermions, the question is where this transition takes place. There is a conjectured upper bound on this number arising from counting degrees of freedom [6], while some lattice studies with fermions in the fundamental representation [7] suggest a value well below this bound. Much model building [8] assumes a value closer to the upper bound. It is clearly important to address this question through further lattice studies, for models with fermions in the fundamental as well as higher representations.

Another critical issue for walking theories is the nature of the bound-state spectrum. The spectrum of these near-conformal theories could be rather different from the QCD spectrum, for example exhibiting an approximate parity doubling. The width to mass ratio of the states in this spectrum could also be rather different from QCD. If a walking gauge theory provides the underlying mechanism for electroweak symmetry breaking, then the properties of the spectrum will determine the parameters of precision electroweak studies such as the S,T, U parameters of Peskin and Takeuchi [9].More importantly, these properties will be explored directly at the LHC.

A wide class of these models involves fermions interacting with multiple gauge groups. In the standard model, the coupling between the strong and the electro-weak gauge groups occurs only through quarks as intermediaries. Indeed, the coupling of quarks with both gauge fields raises several interesting issues, one of which is the fact that the parity breaking in the standard model is only visible because of a misalignment of these gauge groups [10]. But somehow this occurs without introducing a large CP violation in the strong interactions. Lattice motivated non-perturbative arguments have already provided some input to this puzzle by showing that one proposed solution, the vanishing of a single quark mass, may be ill posed due to non-perturbative ambiguities in defining the masses of confined constituents [11].

As the number of gauge groups becomes large, can one make connections to the higher dimensions required in string theories? Can these models mimic gravitational curvature in the extra dimensions as used in models such as in Ref. [12]? As non-perturbative phenomena are crucial here, so is the lattice.

2.2 Unification

Unification schemes based on non-perturbative dynamics with groups larger than the $SU(3)$ of the strong interactions are at the heart of many models for physics beyond the standard model. On a fundamental level there are many questions in all these models. Can these approaches shed light on the origin of particle masses? Many of these models have consequences for the evolution of the early universe. The Higgs and related phenomena are expected to produce phase transitions at extremely high temperatures. Are these in any way related to the baryon number asymmetry seen in today's universe? As the models evolve, it is the lattice that can most definitively resolve these issues. Many subtle theoretical questions arise in these more complex gauge theories. For one example, $SU(5)$ was the first candidate for a unification scheme. With the fermions in the 10 representation, the continuum model is expected to have a discrete Z_3 chiral symmetry. Large N_c connections to supersymmetric theories [13] suggest that this is spontaneously broken, but it is not known if it is broken already at $N_c = 5$ or what the physical consequences of such a breaking would be. Indeed, it is not even known if there exists a lattice formulation that preserves this symmetry.

A grand unified model with gauge group $SO(10)$ and fermions in the 16 representation is frequently discussed. This is particularly intriguing since all anomalies are automatically canceled. This could be important from the lattice viewpoint since anomalies are deeply entwined with doubling problems. This model has a Z_4 discrete chiral symmetry. How is this symmetry realized? Can this be used to justify some version of square root procedure for the lattice determinant to obtain a regularization of the chiral theory?

Parity violation is a property of nature. It is known that variations on QCD can spontaneously break parity [14]. Can a similar phenomenon in a unified model be used to construct a model for the parity violation seen in nature? These are all non-perturbative questions for which the lattice is a most promising tool.

2.3 Supersymmetry

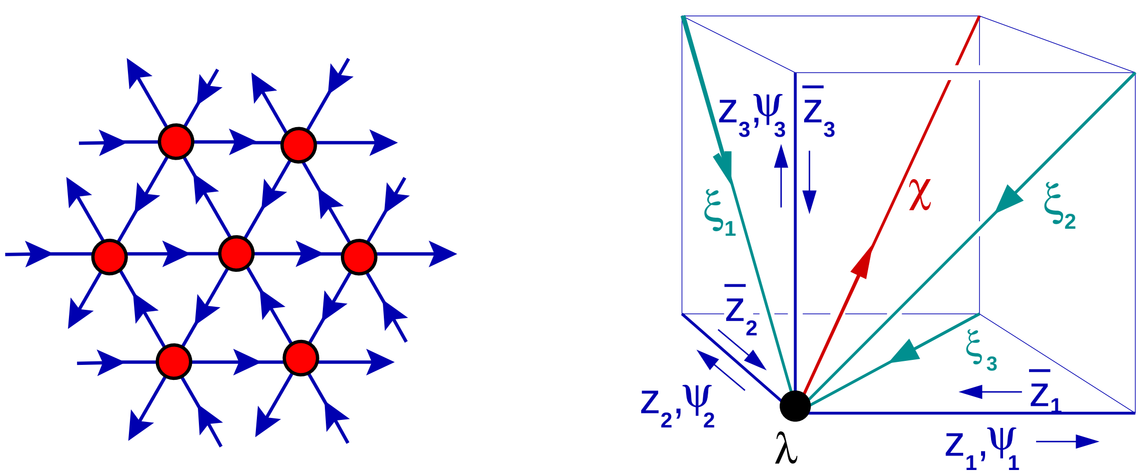

Figure 1: On the left is the lattice for supersymmetric Yang Mills in $d = 2$ with $\mathcal{Q} = 16$ supercharges. On the right is lattice for supersymmetric Yang Mills in $d = 3$ with $\mathcal{Q} = 8$ supercharges. The z_i are bosons, while the other fields are one-component fermions

While supersymmetry may or may not play a role in future particle phenomenology, it does makes rather strong predictions for a variety of quantum field theories. Moreover the study of supersymmetric gauge theory has advanced our understanding of QCD as well as fundamental issues in field theory and in string theory via Maldacena's gauge/string duality. Still there are many unanswered question that only lattice simulations can address as emphasized by Strassler in his excellent review [15].

Placing SUSY field theories on the lattice is not trivial because the full symmetry is an extension of the Poincare group, something which is broken by the lattice itself. There are several ingenious lattice methods that recover supersymmetry as an "accidental" symmetry in the continuum limit.

The goal is to maintain just enough residual supersymmetry to eliminate relevant SUSY breaking operators so that little or no fine tuning is required to reach the target SUSY theory in the continuum limit. $\mathcal{N}=1$ $SU(N_c)$ super Yang Mills is a particularly simple (and a typical) case, where gauge invariance for the adjoint gauge field and chirality via an overlap [16] or domain wall fermion (DWF) construction [17] for the adjoint partners is sufficient to guarantee accidental supersymmetry in the continuum limit. Moreover the Fermionic Pfaffian on the lattice is positive definite so this is a pristine example of a SUSY lattice theory. This theory is predicted to have a discrete Z_{2N_c} chiral symmetry which has been suggested to break spontaneously to a Z_2. This can in principle be tested in lattice simulations. Furthermore, the lattice is flexible enough to explore non-trivial variations for which the structure is unknown.

First generation numerical simulations of the full $\mathcal{N}=1$ $SU(2)$ super Yang Mills theory on a toroidal lattice have already been performed using domain wall fermions [18]. The breaking of the $U(1)_R$ symmetry down to Z_2 is indeed observed. The presence of fractional topological charge on a toroidal lattice was also observed in the quenched theory using the overlap method. This study demonstrates that the lattice can in fact explore the difficult non-perturbative questions regarding this theory, although larger simulations are needed to study the detailed pattern of the $U(1)_R$ symmetry breaking. The calculation of the full spectrum is of great interest since it should expose the inner workings of supersymmetry in a quantitative way. Furthermore, the connection to one flavor QCD is of fundamental interest [19]. There is a remarkable large N proposal by Armoni, Shifman and Veneziano [20], an equivalence between the bosonic subsector of supersymmetric Yang-Mills theory and the charge-conjugation-even bosonic subsector of one flavor QCD. This relation appears to be valid so long as charge conjugation symmetry is unbroken in the latter [21]. This may provide greater understanding about the structure of large N limits via the interrelations to supersymmetric theory. A very broad, and potentially useful class of such equivalences is also being explored by Kovtun, Unsal and Yaffe [22, 23].

A much larger range of SUSY theories are beginning to be considered on the basis of elegant lattice constructions, one using an orbifolding of a supersymmetric matrix model [24] or another based on a discretization of a `twisted` formulation of the supersymmetric theory [25]. These lead to surprising lattice geometries such as that illustrated in Fig. 1, where the fermionic partners are scattered on the lattice in manner reminiscent of staggered fermions but with no unphysical degrees of freedom [24]. In the twisted formulations this connection is explicit - the twisted theories contain multiplets of Kähler-Dirac fields representing the fermions. The equivalence of Kähler-Dirac fields to staggered fermions has been known since the early days of lattice field theory [26]. Algorithmic methods for exploring supersymmetric field theories are in their infancy, but initial attempts show promise.

2.4 String theory and large N_c Yang Mills

Maldacena's so-called AdS/CFT duality hypothesis has dramatically changed the relationship between non-perturbative consequences of Yang-Mills theory and string theory [27]. This not only gives new support to the original conjecture of an exact string equivalence to the QCD gauge theory, but extends it dramatically and suggests new ways to model non-perturbative properties of 4-d Yang Mills theory in the dual extra-dimensional AdS like geometry. For example the String/Gauge duality,

$$\mathcal{N} = 4 \text{ Super Yang Mills} \equiv \text{Super strings in } AdS^5 \times S^5 \ ,$$

identified by Maldacena is conformal and therefore identifies superstrings with a gauge theory which has **neither confinement or narrow flux tubes**. These strings map into a purely Coulombic regime. On the other hand from this example flows a growing class of String/Gauge dualities by breaking the conformal symmetry and some or all super symmetries leading to QCD-like confining theories. Lattice field theory has the opportunity to directly confront and to explore the underlying physics of String/Gauge duality by solving a variety of Yang Mills theories directly in the strong coupling regime. In the large N_c limit for the $SU(N_c)$ gauge group, the dual string becomes non-interacting, thereby exposing the spectrum and scattering of strings in curved backgrounds or, equivalently, the solutions to the 2-d sigma model on the string worldsheet. Thus large N_c provides one direct route to find or solve the string equations in curved background metrics – as yet very poorly understood aspects of string theory.

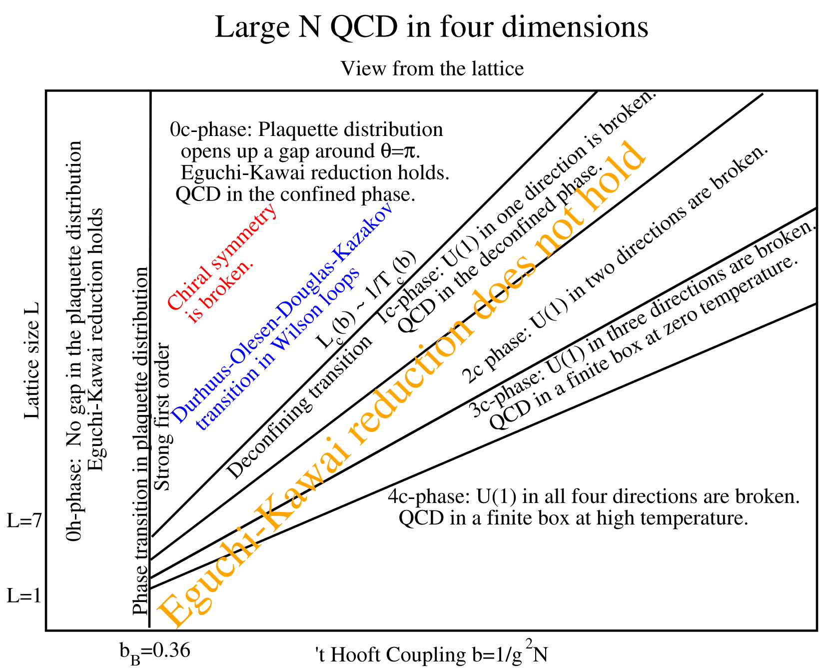

Figure 2: Phase diagram for large N QCD on a 4-d Torus of size L^4 as a function of the inverse 'tHooft coupling $b = 1/g^2N$.

Ironically, from the lattice perspective, the large N_c behavior of QCD itself may provide the technically simplest example of the String/Gauge duality conjecture. The large N_c limit is a semiclassical limit in the sense that root-mean-square fluctuations of operators are suppressed, and distinct quantum theories may behave identically in some subsector. Early work in 1982 by Eguchi and Kawai realized that the large N_c limit of QCD can be replaced by a matrix model so long as certain symmetries are unbroken. This is guaranteed to be true in the strong coupling phase of lattice gauge theory. On the other hand, in the phase continuously connected to the continuum, the full EK reduction fails due to breaking of the center symmetry. Recently, Narayanan and Neuberger [28] realized that the equivalence is valid (for pure Yang-Mills) so long as the size of the lattice is larger than a critical size (see Fig. 2.4). In this regime, the Yang-Mills theory demonstrates volume independence and one obtains infinite volume low temperature results for QCD at $N_c = \infty$. This is a remarkable property of the large N_c limit. Consequently one is now able to compute physical parameters for a limit of the QCD string [29], thus providing precise "experimental" data to challenge various string theoretical formulations of QCD.

More recent work [23] shows that in certain QCD-like theories (Yang-Mills theory with adjoint representation fermions [QCD(Adj)]), endowed with periodic boundary conditions, volume independence does remain valid down to arbitrarily small size, unlike for usual QCD. This is indeed a completely valid continuum reduction, as imagined by EK. The good news is that, in sufficiently large volumes, QCD(Adj) and QCD(AS) (QCD with quarks in the antisymmetric representation, a natural generalization of SU(3) QCD to infinite N_c) have a large N_c "orientifold" equivalence [13], provided charge conjugation symmetry is unbroken in the latter theory [21]. Therefore, via a combined volume independent-orientifold mapping, a well-defined large N_c equivalence exists between QCD(AS) in large, or infinite, volume and QCD(Adj) in arbitrarily small volume. This equivalence should allow a greater understanding of large N_c QCD in infinite volume both analytically and numerically. Again, this field is in its early stages of development, exposing the interrelations between large N lattice calculation and the String/Gauge duality as a fertile ground for future research.

3 Challenges

We see that many fascinating areas need non-perturbative information and the lattice is the primary candidate for such investigations. However, there are several fundamental unsolved problems that must be overcome to extend the applicability of lattice methods for a large class of non-perturbative investigations beyond the standard model. Even modest progress on these difficult challenges are critical to keep the field vibrant while continuing with more conventional methods.

3.1 The sign problem

A frequently occurring problem, also encountered for the QCD at finite chemical potential, is the so-called "sign" problem . Even formulated in Euclidean space, the action may not always lead to a positive semi-definite probability. In most cases this is due to fact that the fermion determinant is not positive, and thus Monte Carlo methods need to be modified. In most severe form this lack of positivity on the lattice is a property of the continuum theory as well. Moving the phase of the determinant from the measure to the observables gives Monte Carlo time growing exponentially with the system size, making many interesting studies impractical. One area where this is particularly important is thermodynamics at high baryon density, where fascinating superconducting phases are expected. This problem is also quite common in current attempts to put supersymmetric theories on the lattice. Another issue lies in the fact that QCD can potentially display a CP violating parameter. This seems to be quite small experimentally, but might not be so in extensions to the standard model. With this present, the sign problem appears. A special case of this is QCD at negative quark masses, where spontaneous CP violation can occur. If such phenomena are important to new physics, finding ways around the sign problems will become essential. Still incremental progress has been made in toy models using the so-called meron algorithm [31], using collective variables to circumvent or greatly reduce the severity of problem. In other cases, such as the Wilson formulation of QCD with an odd number of positive-mass flavors, the sign problem is an artifact of the discretization procedure and can be removed with overlap or domain wall fermions. As difficult as this problem is, it deserves continued research in the absence of a no-go theorem.

3.2 Decay widths and real time evolution

A problem in some way related to the sign problem is difficuties with unstable states and scattering. A characteristics of all current lattice Monte Carlo simulations is the Wick rotation to Euclidean space. From an abstract theoretical point of view this is not important; time evolution is controlled by e^{-Ht} rather than e^{-iHt}, and it is the same Hamiltonian operator in each case. But, as a practical issue, this raises major problems for studies of certain phenomena such as particle decays and real particle scattering. One must do an analytic continuation, and with statistical errors from a Monte Carlo method, this is an ill posed problem. As described in the Nuclear Physics white paper, some progress for low energy states, such as the two pion decay products of a K meson, can be made by finite volume simulations [30]. But in many strongly interacting senarios beyond the standard model, there may be massive states, such as the Higgs itself, that are essential to the physics and that are very unstable and not easily identified by these elegant but limited finite volume methods. We expect searches for new techniques to make this continuation to Minkowski space will represent a major effort in the coming years for all of lattice field theory.

3.3 Chiral gauge theories

Another major unsolved problem in lattice gauge theory is how to formulate a theory where the fermions are coupled to the gauge fields in a non-vectorlike manner [32]. We know that neutrinos are left handed; so, a chiral formulation is essential. The apparent difficulty formulating such theories on the lattice may be a hint at deep physics issues. Are mirror particles to the neutrinos required, perhaps at some large mass? Is the breaking of parity inherent in chiral theories of a spontaneous origin? The chiral coupling to weak interactions in the standard model exploits the Higgs mechanism; is some form of spontaneous breaking a required feature of chiral theories? A non-perturbative formulation is crucial to even framing these questions. There is some progress on these problems. For example Lüsher gives an order by order perturbative argument [33] for anomaly free Abelian chiral theories using the Ginsparg-Wilson relation. He also gives non-perturbative arguments for the special case of chiral theories coupled to compact $U(1)$ gauge theory on the lattice. In the nonabelian case, the overlap with Brillouin-Wigner phase choice of Narayanan and Neuberger [16] appears to be working, but it requires a gauge averaging which destroys locality in the anomalous case. Although not rigorously proven that after gauge averaging the lattice action is local, one can prove the converse, i.e. if anomalies do not cancel it is not local. Much more needs to be understood here.

4 Conclusion

We see that the lattice has the potential to answer non-perturbative questions in quantum field theory that go far beyond the traditional applications to hadronic interactions. These issues are likely to come to the forefront in the LHC era, where a plethora of models will need to be sorted out. The infrastructure created by the SciDAC project will play a major role in allowing these questions to be explored in a timely manner.

However it is important to recognize that the research discussed in this document has an inherently different character from the accompanying white papers on more familiar applications of lattice methods to QCD. Many of the interesting directions discussed here, as well as some important problems in QCD itself,require the invention of new methods. While some other models, such as technicolor, are rather modest extensions of QCD-like theories and are therefore natural candidates for earlier results with higher confidence. The lack of experimental data will make distinguishing lattice artifacts and continuum predictions more challenging and substantially raise the standards for obtaining convincing result. Still this more speculative use of lattice techniques may yield unexpected surprises and insights, potentially leading to the development of powerful new methods along with a deeper insight into the special properties of non-perturbative field theory.

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